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Perturbative SO(10) Grand Unification

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Abstract

We consider a phenomenologically viable $SO(10)$ grand unification model of the unification scale M_G around 10^{16} GeV which reproduces the MSSM at low energy and allows perturbative calculations up to the Planck scale M_P or the string scale M_{st}. Both requirements strongly restrict a choice of Higgs representations in a model. We propose a simple $SO(10)$ model with a set of Higgs representations $\{2 \times \mathbf{10} + \overline{\mathbf{16}} + \mathbf{16} + \mathbf{45}\}$ and show its phenomenological viability. This model can indeed reproduce the low-energy experimental data relating the charged fermion masses and mixings. Neutrino oscillation data can be consistently incorporated in the model, leading to the right-handed neutrino mass scale $M_R \simeq M_G^2/M_P$. Furthermore, there exists a parameter region which results the proton life time consistent with the experimental results.

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1 Introduction

The renormalization group (RG) analysis seems to favor supersymmetric (SUSY) grand unified theories (GUTs) over the non-supersymmetric ones. In particular, with the particle contents of the minimal supersymmetric standard model (MSSM), the three gauge coupling constants converge at the GUT scale $M_G \simeq 2 \times 10^{16}$ GeV [1, 2]. In addition, the recent progress in neutrino physics [3] makes $SO(10)$ GUTs [4] the favorite candidate for grand unified theories because it naturally incorporates the see-saw mechanism [5] that can naturally explain the lightness of the light neutrino masses.

Recently, there has been a lot of attention paid to propose and investigate a "minimal" $SO(10)$ model with so few super multiplets that it can not only fit the current sets of Standard Model data and can even predict a few neutrino-related parameters that experiments are yet to measured accurately. One example of such minimal $SO(10)$ model uses the irreducible representations $\mathbf{10} + \overline{\mathbf{126}} + \mathbf{126}$ in additional to the usual quark and lepton multiplets of three $\mathbf{16}_i$ $(i = 1, 2, 3)$ and only renormalizable operators [6, 7, 8, 9].

One of the main undesirable feature of this approach is that, with the sizes of the super multiplets employed, they contribute such high beta function to the RG evolution such that the GUT gauge coupling constant very quickly blows up to infinity soon after the unification scale, M_G. For example, in the model with a set of Higgs representations $\{\mathbf{10} + \overline{\mathbf{126}} + \mathbf{126} + \mathbf{210}\}$ [10, 11], the coupling constant diverges at $4.2 \times M_G$. While this cannot a priori rule out the model, however it does indicate some unknown physics may take over even before we reach the string scale or the Planck scale. One possibility to explain this run away coupling constant phenomena is to argue that the string scale is actually very near the GUT scale such as in some M-theories [12]. However, this would make the success of GUT-related phenomenology more dubious, since we can not neglect non-renormalizable operators originated from unknown new physics just above the GUT scale. It may be desirable, if achievable, to keep the GUT coupling constant perturbative for at least a couple of order of magnitude before it reaches the (perturbative) string scale $M_{st} \simeq 5 \times 10^{17}$ GeV [13] or the (reduced) Planck scale $M_P \simeq 2.4 \times 10^{18}$ GeV.

On the other hand, the desert scenario associated with the success of the MSSM coupling constant unification dictates that the GUT scale has to be only about two orders of magnitude lower than the Planck scale. It is unavoidable that some higher dimensional operators induced by the higher string scale, M_{st} or the Planck scale M_P, may play some crucial phenomenological role in the analysis of the GUT models. This of course can make the simple GUT models much less predictive. However, it is unnatural also to analyze GUT models pretending that the string or the Planck scales are not out there not too far away. One reasonable strategy to pursue predictability is to use only a minimal set of higher dimensional operators as dictated by the requirement of fitting the low energy phenomenology. This bottom up approach will leave it to the eventual string or Planck scale physics to explain why only these subsets of higher dimensional operators should play important role in the GUT model analysis.

With these perspective in mind, in this paper, we propose a different approach to the $SO(10)$ unification. We pose the question: is it possible to have realistic $SO(10)$ unification with perturbative coupling constant up to the Planck scale (or the string scale)? We require the GUT model to have:
(1) The coupling constant unification similar to that of MSSM. This will require that even

IRREP	$l/2$
10	1
16	2
45	8
54	12
120	28
126	35
210	56

Table 1: List of the Dynkin index for the $SO(10)$ irreducible representations up to the **210** dimensional one

if there is intermediate scales below the GUT scale, it will have to quite close to M_G.
(2) The GUT coupling constant remain perturbative, say $\alpha_G \leq 1$, up to the Planck scale. While it is possible the new physics may come in at the string scale lower than the Planck scale, here, in the first analysis, we use Planck scale because it gives stronger constraint on the beta function of the GUT theory.
(3) The GUT model should fit all known low energy experimental data for the Standard Model parameters including CP violating phase. There is an issue of the role played by the yet undetermined soft SUSY breaking terms. Here we shall assume initially that they play no role in the fit to low energy parameters. It is partly because soft SUSY breaking sector is the most uncertain part of this analysis. It is reasonable to leave them out until it is determined later that they are needed to perfect the model.

2 Perturbative $SO(10)$

The requirement that the $SO(10)$ gauge coupling constant remains perturbative up to M_P imposes severe constraint on the set of matter and Higgs representations we can use. To derive this constraint, note the solution of the (one-loop) RG equation for the unified gauge coupling α_G,

$$\frac{1}{\alpha_G(\mu)} = \frac{1}{\alpha_G(M_G)} - \frac{b}{2\pi} \log\left(\frac{\mu}{M_G}\right) , \tag{1}$$

where $b = -b_{gauge} + b_{matter} + b_{Higgs}$ is the beta function coefficient. Each chiral super multiplet contributes $l/2$ to b, and each vector (gauge) multiplet contributes $3l/2$ where l is the Dynkin index of the irreducible representation listed in Table 1. For $SO(10)$ with three families, $b_{gauge} = 24$ and $b_{matter} = 2 \times 3$, therefore $b = -18 + b_{Higgs}$. If we take the constraint and allows the coupling constant to blow up at $\mu = \Lambda$, namely $1/\alpha_G(\Lambda) = 0$, we obtain

$$b_{Higgs} \leq 18 + \frac{2\pi}{\ln(\frac{\Lambda}{M_G})} \times \frac{1}{\alpha(M_G)} . \tag{2}$$

In MSSM RG analysis, one typically finds $1/\alpha(M_G) \sim 24$. Therefore $b_{Higgs} \leq 49$, if one uses $\Lambda = M_P \simeq 2.4 \times 10^{18}$ GeV, the reduced Planck mass. If one use the stricter condition

that $\alpha(M_P) = 1$, then the constraint becomes $b_{Higgs} \leq 48$ which is about the same as before. Clearly to keep the couplings perturbative, it is necessary to largely reduces the Higgs representations. It is clear from Table 1 that the Higgs representations $\{\overline{\mathbf{126}} + \mathbf{126}\}$ or $\mathbf{210}$ are forbidden to be introduced into a model. Here note that $\mathbf{126}$ is necessary if $\overline{\mathbf{126}}$ is used to break symmetry because of D-flatness condition needed for preserving supersymmetry at the GUT scale.

3 Classifying models

Two main tasks of the Higgs representations are (1) to break the $SO(10)$ gauge symmetry down to the Standard Model one and (2) to give fermion masses and mixings consistent with all the current experimental data. While there are a priori many choices for Higgs representations, we may pick up some Higgs representations to make our model as simple as possible.

For gauge symmetry breaking, the possible choices are:
(a) $\{\mathbf{45}+\mathbf{54}\}$ which contribute $b_{Higgs} = 20$; (b) $\{\overline{\mathbf{16}}+\mathbf{16}+\mathbf{45}\}$ which contributes $b_{Higgs} = 12$; (c) $\{\overline{\mathbf{16}} + \mathbf{16} + \mathbf{54}\}$ which contributes $b_{Higgs} = 16$.
The possibility (a) has been analyzed in the literature before. It certainly achieve the symmetry breaking down to $G_{SM} = SU(3)\times SU(2)\times U(1)$. For (b), a simple superpotential,

$$W = M_{45}\mathbf{45}_H\mathbf{45}_H + M_{16}\overline{\mathbf{16}}_H\mathbf{16}_H + \lambda\overline{\mathbf{16}}_H\mathbf{16}_H\mathbf{45}_H \tag{3}$$

achieves the first task, and one can show that in this case, $\mathbf{45}_H$ can develop VEV that breaks $SO(10) \to G_{2231} = SU(2) \times SU(2) \times SU(3) \times U(1)$ and H_{16} can further break it to G_{SM}. Note that, in this case, since the usual quark and lepton multiplets also belong to a $\mathbf{16}$, it is necessary to impose a global symmetry like R-parity to distinguish between the usual matter and Higgs multiplets. For (c), the superpotential is only

$$W = M_{54}\mathbf{54}_H\mathbf{54}_H + M_{16}\overline{\mathbf{16}}_H\mathbf{16}_H\ . \tag{4}$$

Therefore symmetry breaking is not possible. However, there are still the potential of using the higher dimensional operators to help with symmetry breaking. We shall not treat this more complicated possibility in this manuscript.

For fermions masses, there are lots of possible choices for Higgs representations and the situation can be more complex. We will define our "minimal model" as the one that can accomplish all the above tasks and contributes b_{Higgs} as small as possible.

Considering that top Yukawa coupling is of order one, it is necessary to introduce, at least, one Higgs representation which has a renormalizable Yukawa coupling with $\mathbf{16}$ matters. Although both of $\mathbf{10}$ and $\overline{\mathbf{126}}$ can accomplish this task, $\overline{\mathbf{126}}$ Higgs is forbidden as discussed in the previous section. Thus, we introduce one $\mathbf{10}$ Higgs into our model. Moreover, in order to incorporate Majorana masses of right-handed neutrinos, $\overline{\mathbf{16}}$ Higgs is necessary and the superpotential

$$W = \frac{1}{M_P} Y_{16}^{ij}\mathbf{16}_i\mathbf{16}_j\overline{\mathbf{16}}_H\overline{\mathbf{16}}_H\ , \tag{5}$$

can provide Majorana masses of right-handed neutrinos through VEVs of the MSSM singlet components in $\overline{\mathbf{16}}_H$ and $\mathbf{16}_H$. It leads to a natural scale of the right-handed neutrino mass

as the one derived from the neutrino oscillation data,

$$M_R \simeq \frac{M_G^2}{M_P} \; . \tag{6}$$

Throughout this paper, we assume only these MSSM singlet components of $\overline{\mathbf{16}}_H$ and $\mathbf{16}_H$ develop their VEVs. This assumption is essential to write down the GUT mass matrix relations for the charged fermions (see the next section). Then, the most reasonable choice of the Higgs representations would be $\{\mathbf{10}+\mathbf{16}+\overline{\mathbf{16}}+\mathbf{45}\}$.

With these Higgs representations, the superpotential possibly relevant to the fermion masses is given by (up to dimension 5 terms)

$$W \;=\; Y_{10}^{ij}\mathbf{16}_i\mathbf{16}_j\mathbf{10}_H + \frac{1}{M_P}Y_{45}^{ij}\mathbf{16}_i\mathbf{16}_j\mathbf{10}_H\mathbf{45}_H + \frac{1}{M_P}Y_{\overline{16}}^{ij}\mathbf{16}_i\mathbf{16}_j\overline{\mathbf{16}}_H\overline{\mathbf{16}}_H \; , \tag{7}$$

where the Yukawa coupling matrices Y_{10}, $Y_{\overline{16}}$ are symmetric, while Y_{45} is antisymmetric. Here we have omitted a term proportional to $\mathbf{16}_i\mathbf{16}_j\mathbf{16}_H\mathbf{16}_H$, since this is irrelevant to the fermion mass matrix under the above assumption. One can introduce some global symmetry (Z_N symmetry, for example) to forbid some superpotential terms from the beginning, so that the model becomes simpler. Such global symmetry also plays a crucial role to forbid some dimension five operators (such as $\mathbf{16}_i\mathbf{16}_j\mathbf{16}_k\mathbf{16}_l/M_P$) in the starting Lagrangian, which causes too rapid proton decay.

In the second term in Eq. (7), a product $\mathbf{10}_H\mathbf{45}_H$ plays the same role as a $\mathbf{120}$ Higgs representation. After VEVs of the Higgs doublets in $\mathbf{10}_H$ and $\mathbf{45}_H$ in the $B-L$ direction are developed, the first two terms in Eq. (7) provide Dirac mass matrices of quarks and leptons. Note however that this model so far is obviously unrealistic since it predicts the Kobayashi-Maskawa matrix being unity. This is because the two terms can be factorized by the same $\mathbf{10}_H$ and, as result, the up-type quark mass matrix is proportional to the down-type quark mass matrix. A simple way to ameliorate this problem is to introduce a new $\mathbf{10}$ Higgs and the superpotential such as

$$W = Y_{10}^{ij}\mathbf{16}_i\mathbf{16}_j\mathbf{10}_1 + \frac{1}{M_P}\,Y_{45}^{ij}\mathbf{16}_i\mathbf{16}_j\mathbf{10}_2\mathbf{45}_H \; , \tag{8}$$

where $\mathbf{10}_1$ and $\mathbf{10}_2$ are two Higgs multiplets of $\mathbf{10}$ representation. Here one can again introduce some global symmetry under which $\mathbf{10}_1$, $\mathbf{10}_2$ and $\mathbf{45}$ transform differently, so that the couplings of $\mathbf{10}_1$ and $\mathbf{10}_2$ are arranged as above. In this case, the second term plays the same role of the elementary Higgs of $\mathbf{120}$ representation and thus this system is effectively the same as the one with $\mathbf{10}+\mathbf{120}$ elementary Higgs multiplets. Then, our "minimal model" is defined by the choice of the set of Higgs representations $\{2\times\mathbf{10}+\overline{\mathbf{16}}+\mathbf{16}+\mathbf{45}\}$[6].

4 Fermion mass matrices and low energy data fitting

In the following, we use effective $\mathbf{120}$ Higgs representation in the analysis. The Yukawa couplings relevant to the Dirac mass matrices are given by

$$W = Y_{10}^{ij}\mathbf{16}_i\mathbf{16}_j\mathbf{10}_H + Y_{120}^{ij}\mathbf{16}_i\mathbf{16}_j\mathbf{120}_H \; , \tag{9}$$

[6]The "minimal" models similar to our model has been proposed by many authors [14].

where Y_{10} and Y_{120} are symmetric and anti-symmetric, respectively. Here note that $Y_{120} = Y_{45}\langle \mathbf{45}_H \rangle / M_P$ in the terms of the original superpotential with the VEV of $\mathbf{45}_H$ in the $B-L$ direction. Both of the Higgs multiplets $\mathbf{10}_H$ and $\mathbf{120}_H$ include a pair of Higgs doublets in the MSSM decomposition. At low energy after the GUT symmetry breaking, the superpotential leads to

$$\begin{aligned} W &= (Y_{10}^{ij} H_{10}^u + Y_{120}^{ij} H_{120}^u) u_i^c q_j + (Y_{10}^{ij} H_{10}^d + Y_{120}^{ij} H_{120}^d) d_i^c q_j \\ &+ (Y_{10}^{ij} H_{10}^u - 3 Y_{120}^{ij} H_{120}^u) N_i \ell_j + (Y_{10}^{ij} H_{10}^d - 3 Y_{120}^{ij} H_{120}^d) e_i^c \ell_j \ , \end{aligned} \tag{10}$$

where H_{10} and H_{120} correspond to the Higgs doublets in $\mathbf{10}_H$ an $\mathbf{120}_H$, which originate $\mathbf{10}_1$ and $\mathbf{10}_2$ in the superpotential of Eq. (8). The factor -3 in the lepton sector is the results from the VEV of $\mathbf{45}_H$ in the $B-L$ direction, and plays a crucial role so that unwanted GUT mass relations, $m_e = m_d$ and $m_\mu = m_s$, is corrected.

In order to keep the successful gauge coupling unification, suppose that one pair of Higgs doublets given by a linear combination of $H_{10}^{u,d}$ and $H_{120}^{u,d}$ is light while the other pair is heavy ($\simeq M_G$). The light Higgs doublets are identified as the MSSM Higgs doublets (H_u and H_d) and given by

$$\begin{aligned} H_u &= \widetilde{\alpha}_u H_{10}^u + \widetilde{\beta}_u H_{120}^u \ , \\ H_d &= \widetilde{\alpha}_d H_{10}^d + \widetilde{\beta}_d H_{120}^d \ , \end{aligned} \tag{11}$$

where $\widetilde{\alpha}_{u,d}$ and $\widetilde{\beta}_{u,d}$ denote elements of the unitary matrix which rotate the flavor basis in the original model into the (SUSY) mass eigenstates. Omitting the heavy Higgs mass eigenstates, the low energy superpotential is described by only the light Higgs doublets H_u and H_d such that

$$\begin{aligned} W_Y &= u_i^c \left(\alpha^u Y_{10}^{ij} + \beta^u Y_{120}^{ij} \right) H_u \, q_j + d_i^c \left(\alpha^d Y_{10}^{ij} + \beta^d Y_{120}^{ij} \right) H_d \, q_j \\ &+ N_i \left(\alpha^u Y_{10}^{ij} - 3\beta^u Y_{120}^{ij} \right) H_u \, \ell_j + e_i^c \left(\alpha^d Y_{10}^{ij} - 3\beta^d Y_{120}^{ij} \right) H_d \, \ell_j \ , \end{aligned} \tag{12}$$

where the formulas of the inverse unitary transformation of Eq. (11), $H_{10}^{u,d} = \alpha^{u,d} H_{u,d} + \cdots$ and $H_{120}^{u,d} = \beta^{u,d} H_{u,d} + \cdots$, have been used.

Providing the Higgs VEVs, $\langle H_u \rangle = v \sin\beta$ and $\langle H_d \rangle = v \cos\beta$ with $v \simeq 174$ GeV, the Dirac mass matrices can be read off as

$$\begin{aligned} M_u &= c_{10} M_{10} + c_{120} M_{120} \ , \\ M_d &= M_{10} + M_{120} \ , \\ M_D &= c_{10} M_{10} - 3 c_{120} M_{120} \ , \\ M_e &= M_{10} - 3 M_{120} \ , \end{aligned} \tag{13}$$

where M_u, M_d, M_D and M_e denote up-type quark, down-type quark, neutrino Dirac, charged-lepton mass matrices, respectively. Note that all the mass matrices are described by using only two basic mass matrices, a symmetric M_{10} and an antisymmetric M_{120}, and two complex coefficients c_{10} and c_{120}, which are defined as $M_{10} = Y_{10} \alpha^d v \cos\beta$, $M_{120} = Y_{120} \beta^d v \cos\beta$, $c_{10} = (\alpha^u/\alpha^d) \tan\beta$ and $c_{120} = (\beta^u/\beta^d) \tan\beta$, respectively.

These mass matrix formulas lead to the GUT mass matrix relation among the quark and lepton mass matrices,

$$M_e = c_d \left(M_d + \kappa M_u \right) \ , \tag{14}$$

where

$$c_d = -\frac{3c_{10} + c_{120}}{c_{10} - c_{120}} \, ,$$
$$\kappa = -\frac{4}{3c_{10} + c_{120}} \, . \tag{15}$$

For simplicity, we assume that M_{10} and M_{120} are real and pure imaginary matrices, respectively, and c_{10} and c_{120} are both real. Then, all the Dirac mass matrices becomes hermitian [7] and still CP violating. Note that, according to this assumption, the number of free parameters in our model are reduced into eleven in total; six real parameters in M_{10}, three real parameters in M_{120}, c_{10} and c_{120}. On the other hand, the number of observables we should fit is thirteen; six quark masses, three angles and one CP-phase in the CKM matrix and three charged lepton masses. Thus there are two predictions for observables, whose values have been already known by experiments. Therefore, the data fitting in our model is very non-trivial. In the following analysis, the strange quark mass and the CP-phase in the CKM matrix will be regarded as two predictions in our model (see the following discussion).

Without loss of generality, we can begin with the basis where M_u is real and diagonal, $M_u = D_u$. In this basis, the hermitian matrix M_d can be described as $M_d = V_{CKM} D_d V_{CKM}^\dagger$ by using the CKM matrix V_{CKM} and the real diagonal mass matrix D_d[7]. Considering the basis-independent quantities, $\mathrm{tr}\,(M_e)$, $\mathrm{tr}\,(M_e^2)$ and $\det\,(M_e)$, and eliminating c_d, we obtain two independent equations,

$$\left(\frac{\mathrm{tr}(\widetilde{M_e})}{m_e + m_\mu + m_\tau}\right)^2 = \frac{\mathrm{tr}(\widetilde{M_e}^2)}{m_e^2 + m_\mu^2 + m_\tau^2} \, , \tag{16}$$

$$\left(\frac{\mathrm{tr}(\widetilde{M_e})}{m_e + m_\mu + m_\tau}\right)^3 = \frac{\det(\widetilde{M_e})}{m_e \, m_\mu \, m_\tau} \, , \tag{17}$$

where $\widetilde{M_e} \equiv V_{CKM} \, D_d \, V_{CKM}^\dagger + \kappa D_u$. With input data of six quark masses, three angles and one CP-phase in the CKM matrix and three charged lepton masses, we solve the above equations and determine κ. Using the κ determined, c_d is fixed by

$$c_d = \frac{m_e + m_\mu + m_\tau}{(m_d + m_s + m_b) + \kappa \, (m_u + m_c + m_t)} \, . \tag{18}$$

The original basic mass matrices, M_{10} and M_{120}, are described by

$$M_{10} = \frac{3 + c_d}{4} \, V_{CKM} D_d V_{CKM}^\dagger + \frac{c_d \, \kappa}{4} \, D_u \, , \tag{19}$$

$$M_{120} = \frac{1 - c_d}{4} \, V_{CKM} D_d V_{CKM}^\dagger - \frac{c_d \, \kappa}{4} \, D_u \, . \tag{20}$$

Once the solutions c_d and κ are obtained, M_{10} and M_{120} are completely determined.

Note that it is a very non-trivial problem to find a solution that satisfies both Eqs. (16) and (17) at the same time with only one free parameter κ. In the following analysis, we vary

[7] In general, $M_d = U \, D_d \, U^\dagger$ by using a general unitary matrix $U = e^{i\alpha} e^{i\beta T_3} e^{i\gamma T_8} V_{CKM} e^{i\beta' T_3} e^{i\gamma' T_8}$. In this paper, we adopt the diagonal phases to 0 or π for simplicity.

two input data, the strange quark mass (m_s) and the CP-phase (δ) in the CKM matrix, within their experimental errors, so that both Eqs. (16) and (17) can be satisfied with the same κ value in good accuracy. We can find a consistent solution only if we input special values for m_s and δ. This fact indicates that the input values for m_s and δ we have used in our analysis are two predictions in our model as mentioned above.

Now let us solve the GUT relation and determine c_d and κ. We follow the same strategy in [8]. Since the GUT mass matrix relation is valid only at the GUT scale, we first evolve the data at the weak scale to the ones at the GUT scale with given $\tan\beta$ according to the renormalization group equations (RGEs) and use them as input data at the GUT scale. We take input the absolute values of the fermion masses at M_Z as follows (in GeV):

$$\begin{aligned} m_u &= 0.00233, \quad m_c = 0.677, \quad m_t = 176, \\ m_d &= 0.00469, \quad m_s = 0.0747, \quad m_b = 3.00, \\ m_e &= 0.000487, \quad m_\mu = 0.103, \quad m_\tau = 1.76. \end{aligned} \tag{21}$$

Here the experimental values extrapolated from low energies to M_Z were used [15], and we choose the signs of the input fermion masses as $(m_u, m_c, m_t) = (-, -, +)$ and $(m_d, m_s, m_b) = (-, -, +)$. For the CKM mixing angles and a CP-violating phase in the "standard" parameterization, we input the values measured by experiments as follows:

$$s_{12} = 0.2229, \quad s_{23} = 0.0412, \quad s_{13} = 0.0036, \quad \delta = 57.6^\circ \; . \tag{22}$$

Since it is very difficult to search all the possible parameter region systematically, we present our results for $\tan\beta = 30$. Note that only the case of large $\tan\beta$ can be consistent with our original model, since $Y_{120}^{ij} = Y_{45}^{ij} \langle \mathbf{45} \rangle / M_P \sim 0.01 Y_{45}^{ij}$ and only Y_{10}^{ij} can be of order one. After the RGE running, we obtain the fermion masses and the CKM mixing angles and the CP phase at the GUT scale, and use them as input parameters in order to solve Eqs. (16) and (17). By putting the above data, Eq. (16) gives a solution

$$\kappa = -0.01107011 \cdots \; . \tag{23}$$

On the other hand, from Eq. (17), we obtain the solution

$$\kappa = -0.01107006 \cdots \; . \tag{24}$$

Since these solutions are coincide with each other in good accuracy, we can regard it as the solution we seek. Using Eq. (18) to determine c_d, now we find a solution

$$\begin{aligned} \kappa &= -0.0111 \; , \\ c_d &= -7.89 \; . \end{aligned} \tag{25}$$

The existence of the solution means that our model can reproduce the low energy experimental data for the charged fermion sector.

5 Proton decay

The most characteristic prediction of the SUSY GUTs is the proton decay. Normally in SUSY GUTs the proton decay process through the dimension five operators involving

MSSM matters, mediated by the color triplet Higgsino, turns out to be the dominant decay modes, since the process is suppressed by only a power of the Higgsino mass scale. Experimental lower bound on the proton decay modes $p \to K^+\overline{\nu}$ through the dimension five operators is given by SuperKamiokande (SuperK) [16],

$$\tau(p \to K^+\overline{\nu}) \geq 2.2 \times 10^{33} \text{ [years]} . \tag{26}$$

This is one of the most stringent constraints in construction of phenomenologically viable SUSY GUT models. In fact, the minimal SUSY $SU(5)$ model has been argued to be excluded from the experimental bound together with the requirement of the success of the three gauge coupling unification [18, 17]. However note that the minimal $SU(5)$ model predictions contradicts against the realistic charged fermion mass spectrum, and thus, strictly speaking, the model is ruled out from the beginning. Obviously some extensions of the flavor structures in the model is necessary to accommodate the realistic fermion mass spectrum. On the other hand, knowledge of the flavor structure is essential in order to give definite predictions about the proton decay processes through the dimension five operators. Some models in which flavor structures are extended have been found to be consistent with the experiments in the context of $SU(5)$ models [19, 20] and in $SO(10)$ extensions [21].

As discussed in the previous section, the charged fermion mass matrices have been completely determined in our model. Therefore, we can investigate proton decay rate with only some free parameters. Our discussion follows [22].

The Yukawa interactions of the MSSM matter with the color triplet Higgs induces the following Baryon and Lepton number violating dimension five operator

$$W = C_L^{ijkl} Q^i Q^j Q^k L^l . \tag{27}$$

Here the coefficients are given by the products of the Yukawa coupling matrices and the (effective) color triplet Higgsino mass matrix. In our model, the coefficients are given by the products of two basic Yukawa coupling matrices, Y_{10} and Y_{120}, and the effective 2×2 color triplet Higgsino mass matrix, M_C, such as [11]

$$C_L^{ijkl} = \left(Y_{10}^{ij} ,\ Y_{120}^{ij} \right) \left(M_C^{-1} \right) \left(\begin{array}{c} Y_{10}^{kl} \\ Y_{120}^{kl} \end{array} \right) . \tag{28}$$

As discussed in the previous section, the Yukawa coupling matrices, Y_{10} and Y_{120}, are related to the corresponding mass matrices M_{10} and M_{120} such as

$$\begin{aligned} Y_{10} &= \frac{c_{10}}{\alpha^u v \sin\beta} M_{10} , \\ Y_{120} &= \frac{c_{126}}{\beta^u v \sin\beta} M_{120} , \end{aligned} \tag{29}$$

with $v \simeq 174$ GeV. Here α^u and β^u are the Higgs doublet mixing parameters introduced in the previous section, which are restricted in the range $|\alpha^u|^2 + |\beta^u|^2 \leq 1$. Although these parameters are irrelevant to fit the low energy experimental data of the charged fermion mass matrices, there is a theoretical lower bound on them in order for the resultant Yukawa coupling constant not to exceed the perturbative regime. In order to obtain the most conservative values of the proton decay rate, we make a choice of the Yukawa coupling matrices

as small as possible. In the following analysis, we restrict the region of the parameters in the range $(\alpha^u)^2 + (\beta^u)^2 = 1$ (we assume α^u and β^u real for simplicity). Here we present an example of the Yukawa coupling matrices with fixed $\alpha^u = 0.202$ ($\beta^u = 0.979$),

$$Y_{10} = \left(\begin{array}{ccc} 0.00261 & 0.00485 & -0.00208 \\ 0.00485 & 0.0163 & -0.0414 \\ -0.00208 & -0.0414 & 1.00 \end{array} \right) , \tag{30}$$

$$Y_{120} = \left(\begin{array}{ccc} 0 & -0.0000379\, i & -0.0000379\, i \\ 0.0000379\, i & 0 & -4.61 \times 10^{-6}\, i \\ 0.0000379\, i & 4.61 \times 10^{-6}\, i & 0 \end{array} \right) . \tag{31}$$

Note that the numerical smallness of Y_{120} is a consistency check of our scheme. Since it is a result of the higher dimensional operator, its smallness relative to Y_{10} indicated that it is reasonable to ignore the even higher dimensional operator. However, Y_{120} itself does play important role in fitting low energy data including CP violation.

For the effective color triplet Higgsino mass matrix, we assume degenerate eigenvalues being the GUT scale, $M_G = 2 \times 10^{16}$ GeV, which is necessary to keep the successful gauge coupling unification. Then, in general, we can parameterize the 2×2 mass matrix as

$$M_C = M_G \times U \ , \tag{32}$$

with the unitary matrix,

$$U = e^{i\varphi\sigma_3} \left(\begin{array}{cc} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{array} \right) e^{i\varphi'\sigma_3} \ . \tag{33}$$

Here we omit an over all phase since it is irrelevant to calculations of the proton decay rate. There are four free parameters in total involved in the coefficient C_L^{ijkl}, namely, α^u, φ, φ' and θ. Once these parameters are fixed, C_L^{ijkl} is completely determined.

Through the same numerical analysis as in [22] we can find a parameter region in which the proton life time can be in the range consistent with the SuperK results. In fact, we can find a special colored Higgsino mass matrix that can cancel the proton decay rate through the dominant mode $p \to K^+ \overline{\nu}_\tau$. For example, for the Yukawa coupling matrices of Eqs. (30) and (31), it is found to be ($\tan\beta = 30$)

$$M_C = M_{\rm G} \times \left(\begin{array}{cc} -0.0681\, i & 0.998 \\ -0.998 & 0.0681\, i \end{array} \right) , \tag{34}$$

in other words, $\theta = 1.64$ [rad], $\varphi = 1.57$ [rad], and $\varphi' = 0$ [rad]. With these parameters fixed, the proton life time through the sub-dominant decay modes is estimated as follows:

$$\tau(p \to K^+ \overline{\nu}_e) \ = \ 2.5 \times 10^{35} \text{ [years]} \ , \tag{35}$$

$$\tau(p \to K^+ \overline{\nu}_\mu) \ = \ 4.3 \times 10^{33} \text{ [years]} \ . \tag{36}$$

In our analysis, we have taken the averaged squark mass of the 1st and 2nd generations as $m_{\widetilde{q}} = 10$ TeV and the Wino mass as $M_{\widetilde{W}} = 500$ GeV. These results exceed the current experimental lower bounds. Therefore, our model passes the proton decay constraint and is phenomenologically viable.

6 Neutrino physics

In our model, the right-handed Majorana neutrino mass matrix is generated by Eq. (5) through VEV of $\overline{\mathbf{16}}_H$ in the MSSM singlet direction. Here $Y_{\overline{\mathbf{16}}}$ is the complex symmetric matrix which has twelve free parameters in general and it is nothing to do with charged fermion mass matrices. Therefore, through the see-saw mechanism, there is enough number of free parameters to fit all the current neutrino oscillation data. In other words, there is no prediction for neutrino oscillation physics. However, there is an interesting feature through the see-saw mechanism.

In the basis where M_e is (positive) real and diagonal, the light neutrino mass matrix is given by the see-saw mechanism $M_\nu = M_D^T M_R^{-1} M_D$. M_ν is diagonalized by the Maki-Nakagawa-Sakata (MNS) mixing matrix such as $M_\nu = U_{MNS}^T \mathrm{diag}(m_1, m_2, m_3) U_{MNS}$. The current neutrino oscillation data provide informations (but not complete) for the mixing angles θ_{ij} in the MNS matrix and m_i.

Recall that, as discussed in Sec. 4, all the elements in the neutrino Dirac mass matrix can be determined in our model. Therefore, information of M_R can be extracted through the (inverse) see-saw relation,

$$M_R = M_D \; M_\nu^{-1} \; M_D^T \; , \tag{37}$$

if the neutrino oscillation data are used as inputs. Since the current experimental data are insufficient to fix all the elements in M_ν, M_R can be described as a function of parameters not yet undetermined by experiments, $M_R = M_R(m_\ell, \theta_{13}, \delta, \beta, \gamma)$, where m_ℓ is the lightest mass eigenvalues of the light Majorana neutrino, δ, β and γ are the Dirac CP-phase and the Majorana CP-phases, respectively. Making some assumptions for these free parameters, one can evaluate M_R concretely, and leads to predictions for physics related to the right-handed neutrino mass matrix, such as, the leptogenesis scenario [23]. This direction would be worth investigating. We leave it for future works.

In addition, an order estimation leads to an interesting consequence. In our model, the neutrino Dirac mass matrix is approximately the same as the up-type quark mass matrix, and hence its heaviest eigenvalue is roughly the same as the top quark mass. As already mentioned, the natural scale of the right-handed neutrino mass is $M_R \simeq M_G^2/M_P \simeq 10^{14}$ GeV. Therefore, according to the see-saw mechanism, we find the heaviest mass eigenvalue of the light neutrinos being of order 0.1 eV. Interestingly, this value is close to $\sqrt{\Delta m_\oplus^2}$, where $\Delta m_\oplus^2 \simeq 2.1 \times 10^{-3}$ eV^2 is the atmospheric neutrino oscillation data [3]. This result indicates that our model prefers the hierarchical case to the degenerate case for the light neutrino mass spectrum.

7 Conclusion

Discovery of the neutrino masses and mixings has made $SO(10)$ GUT models the favorite candidate as new physics. Lots of $SO(10)$ GUT models have been intensively discussed. There are *a priori* many choices for the Higgs representations to be introduced into a model. We have imposed the requirement that the GUT gauge coupling should remain to be perturbative up to the (reduced) Planck scale or the string scale at which further new physics including quantum gravity takes over. This requirement has been found to

be strong enough to forbids Higgs representations higher than **126** dimension. We have proposed a simple $SO(10)$ model with a set of Higgs representations $\{2\times\mathbf{10}+\overline{\mathbf{16}}+\mathbf{16}+\mathbf{45}\}$, which can satisfy the requirement and gives the beta function coefficient of the GUT gauge coupling as small as possible. It has been shown that the model is phenomenologically viable, namely all the charged fermion masses and mixings have been reproduced. In this realistic Yukawa couplings, the most stringent proton decay processes has been suppressed. Also, the model can reproduce the current neutrino oscillation data with the right-handed neutrino mass being of order $M_R \simeq M_G^2/M_P$.

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